Transfer Functions

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Purpose:

Identify and verify circuit transfer functions and practice transfer function design.

Equipment Required:

- 1 - Agilent 54600B Oscilloscope
- 1 - Agilent 34401A Digital Multimeter
- 1 - Agilent 33120A Function Generator
- 1 - Protoboard
- 1 - 1-kΩ Resistor
- 1 - 0.01-µF Capacitor

Various Passive Components

Prelab:

Review the sections in Ch. 9[*] covering pole-zero diagrams and the sections in Chs. 10 and 11 covering transfer functions and transfer function design.

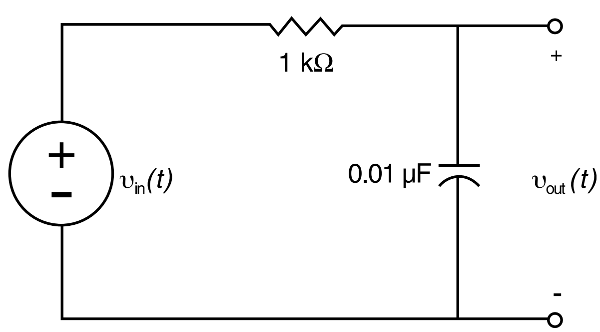

Figure 1

1. Transfer Function Analysis

   The circuit of Fig. 1 is a first-order RC circuit with the output $v_{out}(t)$ taken across the capacitor. The input is the unit step function $v_{in}(t) = u(t)$.

   a. Determine the circuit voltage transfer function $T_V(s)$, the step response transform G(s) and the output waveform g(t) for $v_{out}(t)$ as indicated in Fig. 1.

b. Enter the circuit of Fig. 1 into a CCA program and determine the step response by performing a transient analysis. Plot the results and tape the output into your lab journal.

2. Transfer Function Design

a. Use Table 1 to determine your assigned design project by adding your student number to that of your partner. The last digit of this sum determines the project.

b. The input to the circuit will be a unit step function $v_{in}(t) = u(t)$. Find the step response transform G(s) and the step response waveform g(t) for your assigned transfer function $T_V(s)$.

c. Design a circuit that will realize the voltage transfer function $T_V(s)$ associated with your assigned project. Use the standard component values available in your laboratory. Fully document your design.

Last Digit	0,9	1,8	2,7	3,6	4,5
Voltage Transfer Function $T_V(s)$	$\frac{10000}{s+10000}$	$\frac{s}{s+10000}$	$\frac{s}{s+20000}$	$\frac{20000}{s+20000}$	$\frac{30000}{s+30000}$

Table 1

d. Use transient analysis in a CCA program to verify that your circuit design produces the step response waveform g(t) found in step 2b.

Procedure

1. Transfer Function Analysis

a. Connect the circuit of Fig. 1. Set the output of the function generator to produce a square wave, 1 V_{PP} and a DC offset of +0.5 V_{avg}. You will need to determine an appropriate frequency so that the circuit reaches its final value before the source changes state back to 0 V. The scope should be triggered on the rising edge of the square wave.

b. On a half page in your journal, sketch the waveform displayed on the scope. Label the horizontal and vertical axes with the units and scales.

2. Transfer Function Design

   a. Construct your circuit design that realizes the voltage transfer function $T_v(s)$ assigned in the Prelab. The source $v_{in}(t)$ is a square wave with 1 V_{PP} and a DC offset V_{avg} of 0.5 V. You will need to determine an appropriate frequency to ensure the circuit will have time to reach its final value.

   b. On a half page in your journal, sketch the waveform displayed on the scope. Label the horizontal and vertical axes with the units and scales.

Conclusion

For the step function u(t), all of the activity occurs at t = 0 when the function changes from a value of 0 for all t < 0, to a value of 1 for all t > 0. In practice this singular event is difficult to capture and record. Our approach uses a square wave that, in effect, repeats the step function over and over again. We ignore the square wave transition from a value of 1 back to 0, and concentrate our efforts on recording the positive-going transition.

The step response for the circuit of Fig. 1 is of the form:

$$g(t) = \left[A + Be^{-t/T}\right]u(t) \qquad \textbf{(1)}$$

The initial value of g(t), at t = 0+ is given by:

$$g(0) = A + B \qquad \textbf{(2)}$$

The final value of g(t) is found by letting time t approach infinity. When t reaches infinity, Eq. 1 reduces to:

$$g(\infty) = A \qquad \textbf{(3)}$$

Equation 3 can be applied to the data collected in this lab exercise only if the half-period of the input waveform is greater than five time constants of the circuit, $5T_C$. This requirement on the period of the input waveform can be written as:

$$\frac{To}{2} \geq 5T_c \qquad \textbf{(4)}$$

If the condition specified in Eq. 4 is satisfied, the final value given in Eq. 3 can be determined from the sketch of the response waveform after an interval of time $t > 5T_c$.

1. Transfer Function Analysis

From the sketch of the response waveform drawn in Procedure 1b, determine the values of A, B, and T_c. (the "Exponential Waveforms" experiment provides details on determining the time constant from a sketch of the circuit response.) Use these experimental values for A, B and T_c and Eq. 5 to determine the realized transfer function $T_V(s)$ for the circuit of Fig. 1.

$$
\begin{aligned}
T_V(s) &= sG(s) = s\mathscr{L}g(t) \\
&= s\mathscr{L}[A+Be^{-t/Tc}] \\
&= s\left[\frac{A}{s} + \frac{B}{s+1/T_c}\right] \qquad (5) \\
&= \frac{(A+B)s + A/T_c}{s+1/T_c}
\end{aligned}
$$

Does the transfer function derived from the empirical data agree with the transfer function computed in Prelab 1a? Plot a pole-zero diagram for this circuit.

2. Transfer Function Design

From the sketch of the response waveform drawn in Procedure 2b, determine the values of A, B, and T_c. Use these values to determine $T_V(s)$, and compare this result with your assigned transfer function listed in Table 1. Does this comparison support the claim that your circuit design realizes your assigned transfer function? Plot a pole-zero diagram for this circuit.

* Roland E. Thomas and Albert J. Rosa, The Analysis and Design of Linear Circuits, Prentice Hall, (New Jersey, 1994)